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Definitions.

To define, as the word is ordinarily used in elementary mathematics, is to state precisely the meaning of, or to describe the nature and certain determining properties of the idea defined. To be a good definition the description must be in terms of simpler ideas than the thing defined, and must be sufficiently comprehensive to include every member of its class. It must be ***inclusive,*** in that it includes all of the class defined, and ***exclusive,*** in that it excludes all others. It is therefore a statement or series of statements of which the converse is also true, otherwise it fails in this exclusiveness. It would be true to say that a horse is a quadruped, but it would not define a horse, as all quadrupeds are not horses.

Definitions have a twofold purpose; first, to give the exact meaning of the idea defined in an intelligible form, and second, to give in concise form the determining properties, so that they may be easily held or remembered.

It is apparent that, in general, the more elementary the idea the harder it is to define, since it is harder to find still simpler ideas in terms of which to express its properties. This is very true in arithmetic, for here the ideas of unit, number, etc., are so fundamental that it is almost impossible to find definitions that serve either of the two purposes for which they are made.

In all probability no child ever gets the idea of number from a definition of it. Pupils for the most part come to school with

ideas of the integers at least, and the number concept enlarges with every new type of number encountered. In view of these facts definitions should be used very sparingly in elementary arithmetic. In fact they should be used only where they serve at least one of their two purposes.

If one takes up some of the arithmetics in common use he will find that an attempt is made to define most of the fundamental ideas. And such attempts as some are! The very subject which above others should lead to habits of accurate thinking seems to be the occasion for a display of the very opposite on the part of many writers on elementary mathematics, and particularly on arithmetic.

The definition that "Arithmetic is the *science* of numbers and the *art of* their computation" which is sometimes given would hardly satisfy either of the two purposes of a definition given above. The terms science and art are not comprehensible to the student of elementary arithmetic.

Number is an idea and is essentially abstract, a fact too often lost sight of. Some numbers have physical realities corresponding to them, but the numbers themselves are abstract. In view of this, and of the fact that the converse of a definition must be true the nonsense of the definitions of unit and number often given, even in some of our most recent arithmetics, is apparent. To say that "unit is a single thing" and that "A number is a unit or collection of units" comes very near the limit of absurdity even though we add to the end of the latter "of the same kind."

Another definition, which seems to be a relic of earlier stages of civilization when the race knew integers only, is that of multiplication as "a short process of repeated addition" or perhaps the more recent but practically equivalent one that "Multiplication is the process of taking one number as many times as there are units in another" which does not work very well when the multiplier is a fraction. Other examples might be given but these few will suffice to show that much defining is out of place in elementary arithmetic and such types are out of place anywhere.